CHAPTER 3

VIBRATION THEORY

- Single Degree of Freedom Systems (SDOF)
- Stiffness
- Member Stiffness
- Natural Period
- Natural Frequency
- Angular Natural Frequency
- Structural Damping
- Critical Damping and Damping Ratio
- Multiple Degree of Freedom Systems (MDOF)
- Response Spectra
- Normalized Response Spectra
- Base Shear
- Example 3.1
- Example 3.2
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SINGLE DEGREE OF FREEDOM SYSTEMS (SDOF)

Many simple structures can be idealized as a concentrated or lumped mass, ***m***, supported by a massless structure with stiffness, ***k***, in the lateral direction. A one story building or structural frame as shown in Fig 3.1 has a heavy and stiff/rigid roof. When this building is subjected to lateral load, ***F***, it has only one degree of dynamic freedom, the lateral sway or displacement, ***Δ***, as indicated on the figure. An equivalent dynamic model of this building consists of a single column with equivalent stiffness, *k*, supporting a lumped mass of magnitude, ***m***.

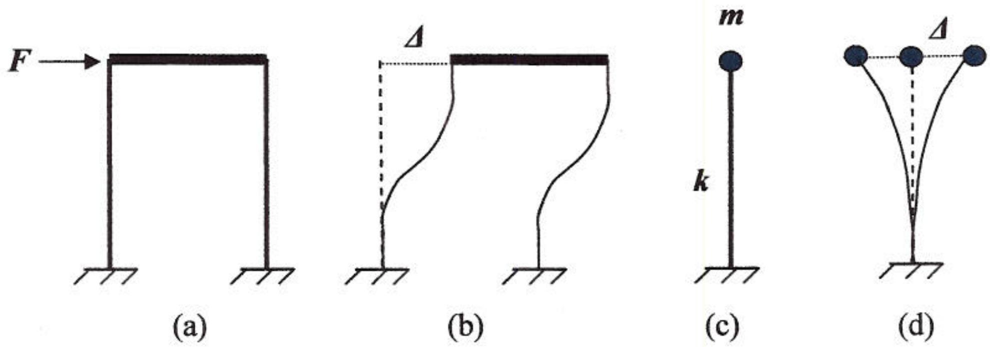


Fig. 3.1 Single Degree of Freedom Systems

STIFFNESS, *K*

Stiffness is the force required to produce a unit displacement in the direction of the force (kips/in, lbs/in).

$$K = F/\Delta$$

When there are multiple columns (members with individual stiffness), the total equivalent stiffness of the SDOF system can be summed from individual member stiffness either in parallel or in series or a combination thereof.

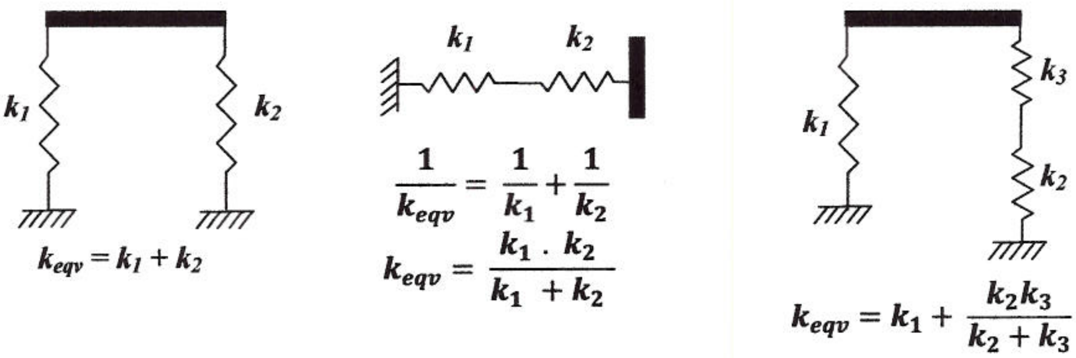


Fig. 3.2 In-parallel and In-Series Equivalent Stiffness

The inverse of stiffness is flexibility. Thus, it is the deflection produced by a unit force (in/kips, in/lbs).

MEMBER STIFFNESS (MEMBER RIGIDITY)

The member stiffness is a function of the length of the member, ***L***, second moment of inertia, ***I***, Material's Young modulus of elasticity, ***E***, Members' cross sectional area, ***A*** (only in the case of axial stiffness), and ends condition (free, pinned, fixed). Member stiffness is also termed member rigidity. **Table 3.1** summarizes stiffness expressions for different end conditions. The member's maximum deflection equations (Force/stiffness) are also shown in the Table.

$$stiffness\ (rigidity) = 1\ /\ deflection$$

NATURAL PERIOD

If the mass of the SDOF system shown on Fig 3.1.d is subjected to an initial displacement and then released, free vibration occurs about the static position producing a harmonic sinusoidal wave (Fig. 3.3). The time required to compete a full cycle of vibration is the natural period of vibration, ***T***. It is the time between two successive peaks or valleys as shown on Fig. 3.3. Natural period are sometimes referred to as the *fundamental period.*

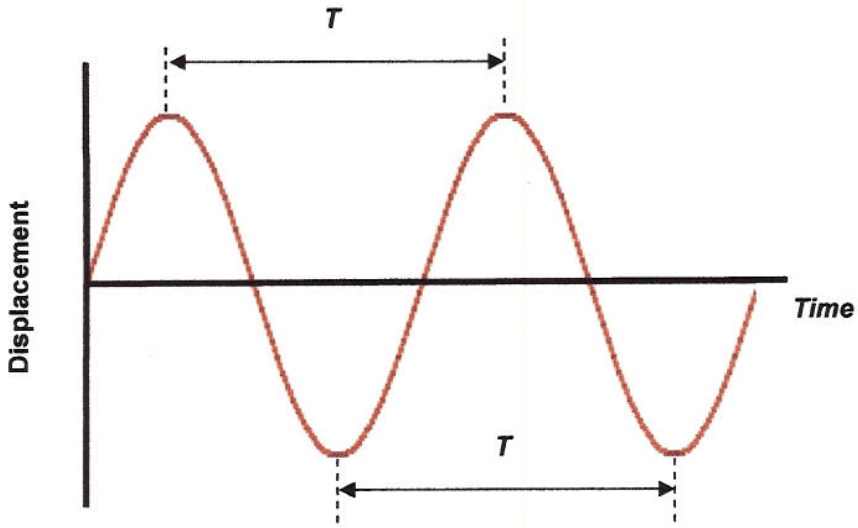


Fig. 3.3 Natural Period of Vibration

Natural period of vibration, T, in sec.:

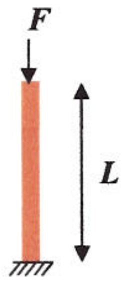
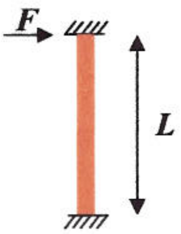
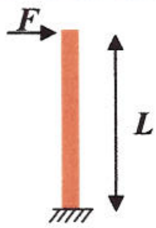
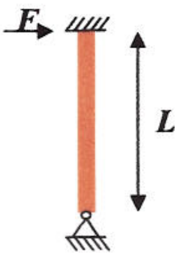
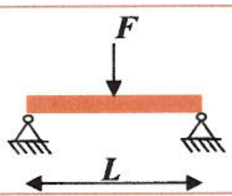
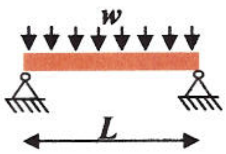
$$\boxed{T = 2\pi\sqrt{m/k}} \quad \text{(sec)}$$

Where $m = W/g$ W = weight of the structure

g = gravitational acceleration = 32.2 ft/sec^2 = 386 in/sec^2

- The form of this expression indicates that the natural period increases as the mass of the system increases. The natural period also increases as the stiffness of the system decreases.

TABLE 3.1 Member Stiffness for Different Boundary Conditions

MEMBER	*STIFFNESS*	*DEFLECTION*
	$\frac{EA}{L}$	$\frac{FL}{EA}$
	$\frac{12EI}{L^3}$	$\frac{FL^3}{12EI}$
	$\frac{3EI}{L^3}$	$\frac{FL^3}{12EI}$
	$\frac{3EI}{L^3}$	$\frac{FL^3}{12EI}$
	$\frac{48EI}{L^3}$	$\frac{FL^3}{48EI}$
	$\frac{384EI}{5L^3}$	$\frac{5wL^4}{384EI}$

Example 3.1:

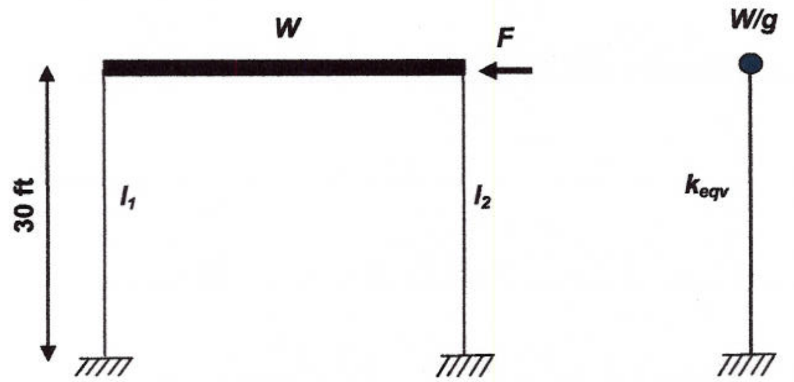
Figure shows a SDOF system, and the Design Response Spectrum where:

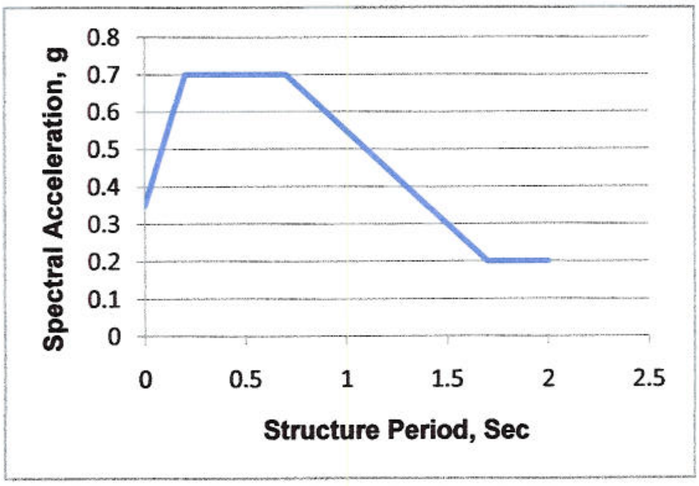
W = 860 kips

E = 29,000 ksi

$I_1 = I_2$ = 22000 k/in

S = 2800 in^3





Design Response Spectrum

Determine Equivalent Stiffness of the SDOF system?

Structure shown can be assumed as SDOF system with lumped mass, m = W/g, and equivalent stiffness k_{eqv}.

$k_{eqv} = 12EI_1/H^3 + 12EI_2/H^3 = 12*29000*22000/(30*12)^3 + 12*29000*22000/(30*12)^3$ = 328 k/in.

Determine natural period and frequency of the SDOF system?

Natural period, $T = 2\pi \sqrt{(W/k_{eqv}.g)} = 2*3.14*\sqrt{(860/328.386)} = 0.52$ sec.

Frequency, $f = 1/T = 1/0.52 = 1.93$ Hz.

Determine Base shear V of the SDOF system?

From design response spectrum shown, for T=0.52 sec., $S_a = 0.7g$

Base Shear, $V = m*S_a = W*S_a/g = 860*0.7g / g = 602$ kips

Determine lateral deflection Δ of the SDOF system?

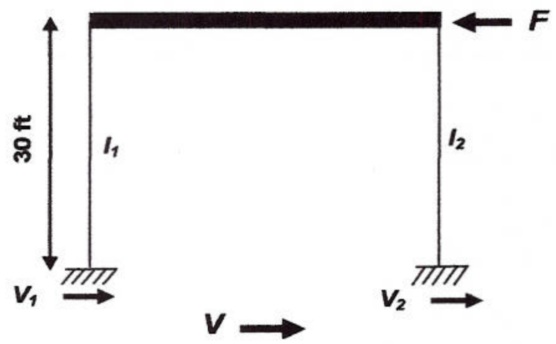
The total base shear V determined above (595 k) equals the lateral force, ***F***, applied at the lumped mass.

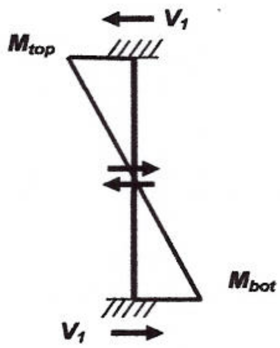
Deflection, $\Delta = F/k_{eqv} = 602/328 = 1.83$ in.

Determine shear force in Column 1?

Base shear will be divided between columns based on their relative stiffness. Since $I_1 = I_2$. Thus $V_1 = V_2 = 602/2 = 301$ kips.

Determine maximum bending moment in Column 1?





$M_{top} = M_{bot} = V_1* H/2 = 301*30/2 = 4515$ k.ft

Determine maximum bending stress in Column 1?

Bending Stress at bottom (or top), $\sigma = M_{bot}/S = 4515/2800 = 19.4$ ksi

Example 3.2:

Figure shows two SDOF systems, and their Design Response Spectrum where:

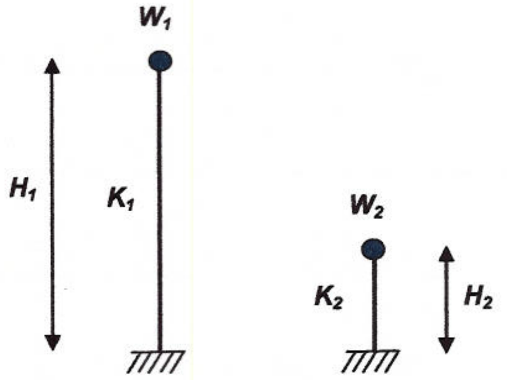
$W_1 = 2\ W_2$

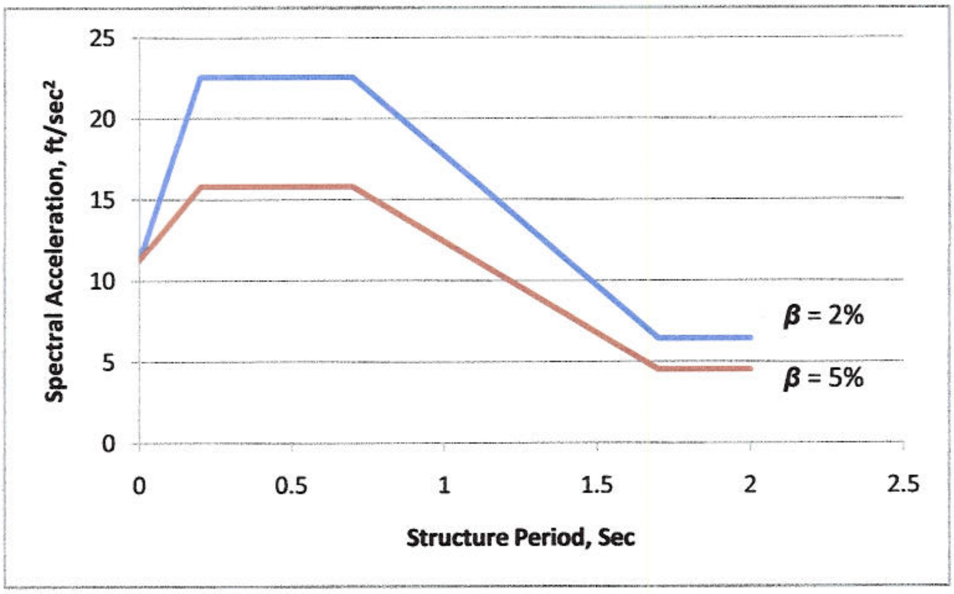
$E_1 = E_2 = E$

$I_1 = 4\ I_2$

$H_1 = 3\ H_2$

$\beta_1 = 5\%$ $\quad\quad$ $\beta_2 = 2\%$





Design Response Spectrum

Determine the stiffness ratio of SDOF 2 to SDOF 1 ?

$K_2 = 3*E*I_2/ (H_2)^3$

$K_1 = 3*E*I_1/(H_1)^3 = 3*E*4*I_2/ (3*H_2)^3 = 3*4*27*E*I_2/ (27*H_2^3) = 0.44\ K_2$

$K_2 = 1/0.44\ K_1 = 2.25\ K_1$

(The shorter the structure, the stiffer it gets)

Determine the natural period ratio of SDOF 2 to SDOF 1 ?

$T_1 = 2\pi \sqrt{}(W_1/k_1.g) = 2\pi \sqrt{}(2*W_2/(0.44*k_2.g)) = 2\pi * \sqrt{}(2*2.25) * \sqrt{}(W_2/k_2.g)$

$T_2 = 2\pi \sqrt{}(W_2/k_2.g)$

$T_1 = \sqrt{}(2*2.25) * T_2 = 2.21 * T_2$ $\qquad T_2 = 0.47 * T_1$

(the taller the structure, the longer the natural period)

Determine the base shear ratio of SDOF 2 to SDOF 1 ?

Assume $T_1 = 1.0$ sec

From Design Response Spectrum

SDOF 1, $\beta_1 = 2\%$ $\qquad T_1 = 1.0$ sec, $\quad S_{a1} =$ 17.6 ft/sec^2

SDOF 2, $\beta_1 = 5\%$ $\qquad T_2 = 0.47$ sec, $\quad S_{a2} =$ 15.8 ft/sec^2

$V_1 = W_1 * S_{a1} /g = W_1 * 17.6 /32.2 = 0.55 * W_1 = 0.55 * 2 * W_2 = 1.1 * W_2$

$V_2 = W_2 * S_{a2} /g = W_2 * 15.8 /32.2 = 0.49 * W_2$

$V_2 / V_1 = 0.49 / 1.1 = 0.45$ $\qquad V_2 = 0.45\ V_1$

(generally, the smaller the mass, the less base shear)

MULITPLE CHOICE QUESTIONS

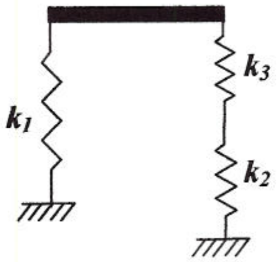
3.1 Member's rigidity can be best described as?

A the inverse of member's deflection
B the inverse of member's stiffness
C the inverse of member's ductility
D the inverse of member's damping

3.2 For a SDOF system, when the mass, *m*, increases, what is the effect on natural period, *T*, and base shear, *V*?

A *T* decreases and *V* increases
B *T* decreases and *V* decreases
C *T* increases and *V* increases
D *T* increases and *V* decreases

3.3 Determine the equivalent stiffness of the SDOF system shown in the figure? Where k_1 = 80 kips/in, k_2 = 60 kips/in, and k_3= 40 kips/in.



A 27 kips/in
B 34 kips/in
C 104 kips/in
D 180 kips/in

3.4 What is the spectral velocity of a single degree of freedom system?

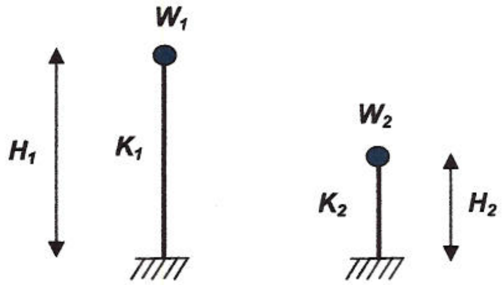
A the minimum velocity experienced by the structure due to a specific ground motion
B the average velocity experienced by the structure due to a specific ground motion
C the maximum acceleration experienced by the structure due to a specific ground motion multiplied times angular frequency
D the maximum displacement experienced by the structure due to a specific ground motion multiplied times angular frequency

3.5 Maximum acceleration experienced by the building from a specific ground motion is defined as?

A peak ground acceleration

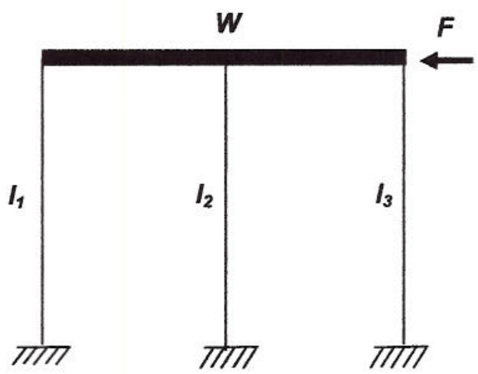
B dynamic acceleration
C spectral acceleration
D maximum building velocity relative to ground acceleration

3.6 Determine the natural period of SDOF 2?
$W_1 = W_2 = W$; $E_1 = E_2 = E$; $I_1 = 2I_2$; $H_1 = 2H_2$



A 0.50 T_1
B 0.85 T_1
C 2.50 T_1
D 3.50 T_1

3.7 For the structure shown subjected to lateral force, *F*, determine the ratio of shear forces distributed to the three columns, respectively? The three columns have the same height, modulus of elasticity and boundary conditions. Assume column's second moment of inertia are distributed as $4I_1 = 2I_2 = I_3$.



A 1/7 *F*, 2/7 *F*, 4/7 *F*
B 4/7 *F*, 2/7 *F*, 1/7 *F*
C 1/3 *F*, 1/3 *F*, 1/3 *F*
D -4/6 *F*, -2/6 *F*, 1.0 *F*

3.8 Natural period, *T*, of a SDOF system is determined from:

A the structure response spectrum
B equivalency to the system's linear frequency
C the inverse of the system's angular frequency
D system's mass and stiffness

3.9 What is the damping of an oscillating SDOF system?

A shortest time between successive cycles of vibration
B energy modification response factor

C rate of change of displacement amplitude
D decay of vibration amplitude with time

3.10 For a Multiple Degrees Of Freedom system, MDOF, the term "higher modes" refers to?

A modes of vibration with the longest periods
B modes of vibration with the shortest periods
C modes of vibration with the shortest frequencies
D modes of vibration with the highest participation factor

3.11 Critical damping of an oscillating harmonic system can be best described as the ....?

A damping to bring the harmonic system to static position in the shortest possible time
B ratio of actual damping to critical mass of the system
C decay of vibration amplitude with time
D factor of vibration underdamped amplitude to overdamped amplitude

3.12 Determine the spectral displacement, S_d, of a SDOF system that has natural period of 0.5 sec, and a spectral acceleration, S_a, of 0.75g?

A 1.54 in.
B 1.84 in.
C 2.94 in.
D 4.75 in.

3.13 Natural period of a SDOF system can be best described as?

A the fundamental period of vibration
B the inverse of natural frequency
C the time between two successive peaks or valleys
D all of the above descriptions

3.14 The response spectrum is best described as ....

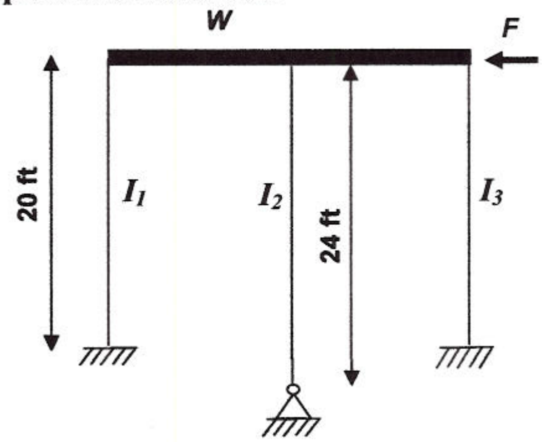
A a graph of the maximum responses of SDOF systems to a specified excitation
B the maximum response by a SDOF system to a specified excitation
C a collection of several response spectra
D the maximum ground response of SDOF system to a specified excitation

3.15 The dynamic response of a MDOF system can be best represented by ....

A the fundamental mode of vibration
B sufficient modes of vibration that contribute more than 90% of the participating mass
C the square root of sum of squares of the first three modes of vibration
D all of the above descriptions

Refer to the building shown in Figure for questions 3.16 to 3.20

W = 1000 kips
E = 29000 ksi
$I_1 = I_3$ = 12000 in^4
I_2 = 15000 in^4
S_a = 0.75g



3.16 Determine the equivalent stiffness of the SDOF system?

A 360 k/in
B 650 k/in
C 660 k/in
D 825 k/in

3.17 Determine the natural period of the SDOF system?

A 0.35 sec
B 0.39 sec
C 0.40 sec
D 0.54 sec

3.18 Determine the lateral deflection of the SDOF system ?

A 0.91 in
B 1.14 in
C 1.16 in
D 2.10 in

3.19 Determine the shear in Column 2 of the SDOF system ?

A 62 *kips*
B 249 *kips*
C 344 *kips*
D 688 *kips*

3.20 Determine the flexural stress in Column 2 of the SDOF system, given sectional modulus, $S = 1000$ in^3 ?

A 12 ksi
B 16 ksi
C 18 ksi
D 36 ksi